Name (IN CAPITALS): ____________________________

Instructor and section number or class time: ________________________

Math 10560 Exam 3
Apr. 19, 2022.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
Total	________

Multiple Choice

1.(6pts) Determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{2}{3}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{n^2+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2}$

2.(6pts) Which one of the following series is conditionally convergent?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{2^n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^5+1}}$

3.(6pts) Which of the power series given below is a power series representation of the function

$$f(x) = x\cos(\sqrt{x})$$

centered at 0?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{(2n+1)!}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(2n+1)!}$.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$.

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(2n)!}$.

4.(6pts) Consider the function $f(x)$ defined as

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{2n}}{2^n n!}, \qquad -\infty < x < \infty.$$

Which of the following statements is true?

(a) $\displaystyle\int f(x)\,dx = C + \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^{2n+1}}{2^n (2n+1)!}$

(b) $\displaystyle f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^{2n-1}}{2^n n!}$

(c) $f'(2) = -1$

(d) $f(2) = 0$

(e) $\displaystyle f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^{2n-1}}{2^{n-1} (n-1)!}$

5.(6pts) Use a well known power series to find the sum of the following series

$$\sum_{n=0}^{\infty} \frac{3^n \pi^n}{n!}$$

(a) $\cos 3$ (b) 1 (c) $e^{3\pi}$ (d) -1 (e) 0

6.(6pts) Consider the following series.

$$(I) \quad \sum_{n=1}^{\infty} \left(\frac{n+1}{4n-1}\right)^n \qquad\qquad (II) \quad \sum_{n=1}^{\infty} \frac{3^n}{(n-1)!}.$$

Which of the following is **<u>true</u>**?

(a) (I) diverges while (II) converges.

(b) The ratio test is inconclusive on (II).

(c) Both of the series diverge.

(d) (I) converges while (II) diverges.

(e) Both of the series converge.

 Initials: ____________

7.(6pts) The degree 3 Taylor polynomial of

$$f(x) = \ln(x)$$

centered at $a = 2$ is given by:

(a) $T_3(x) = \ln(2) + \frac{x-2}{2} - \frac{(x-2)^2}{4} + \frac{(x-2)^3}{4}$

(b) $T_3(x) = \ln(2) + \frac{x-2}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24}$

(c) $T_3(x) = \ln(2) + \frac{x}{2} - \frac{x^2}{4} + \frac{x^3}{4}$

(d) $T_3(x) = 2 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24}$

(e) $T_3(x) = 1 + \frac{x-2}{2} - \frac{(x-2)^2}{4} + \frac{(x-2)^3}{4}$

8.(6pts) Which of the following statements are true about the series

$$\sum_{1}^{\infty} \frac{n^2+1}{n^5-n}?$$

Note (just in case you haven't encountered the verb) To deduce means to arrive at (a fact or a conclusion) by reasoning or to draw as a logical conclusion.

I. One can deduce that this series converges by observing that $\lim_{n\to\infty} \frac{n^2+1}{n^5-n} = 0$.

II. One can deduce that this series converges using the Ratio Test.

III. One can deduce that this series converges using the Limit Comparison Test, comparing with the p-series $\sum_{1}^{\infty} \frac{1}{n^3}$.

(a) I, III only (b) II, III only (c) I, II only

(d) III only (e) None

9.(6pts) Find a power series representation for the function

$$\frac{2}{(1-x)^2}$$

in the interval $(-1, 1)$.
(Hint: Differentiating a well-known power series may help).

(a) $\sum_{n=1}^{\infty} 2nx^{n-1}$

(b) $\sum_{n=1}^{\infty}(-1)^n 2nx^{n-1}$

(c) $\sum_{n=0}^{\infty} \frac{2x^{n+1}}{n}$

(d) $\sum_{n=0}^{\infty} \frac{2(-1)^n x^{n+1}}{n+1}$

(e) $\sum_{n=1}^{\infty} 2^n nx^{n-1}$

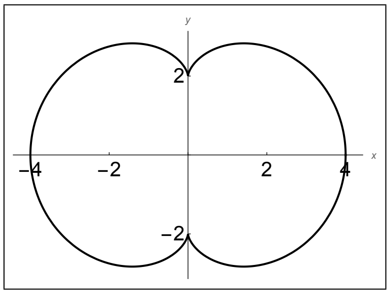
Initials: _____________

10.(6pts) Which of the following is a graph of the parametric curve defined by

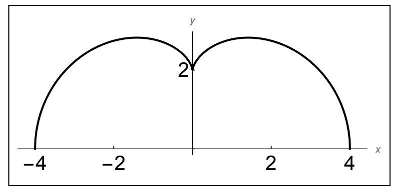
$$x = 3\cos(t) + \cos(3t), \qquad y = 3\sin(t) - \sin(3t)$$

for $0 \le t \le 2\pi$?

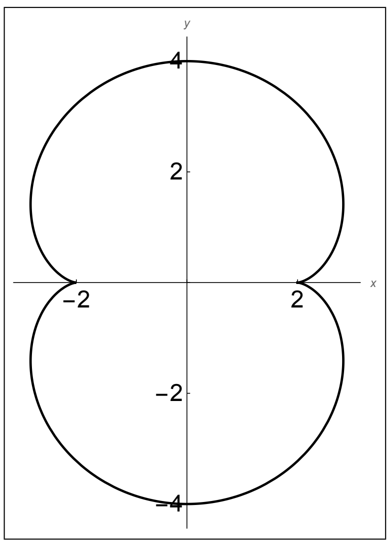
(a)



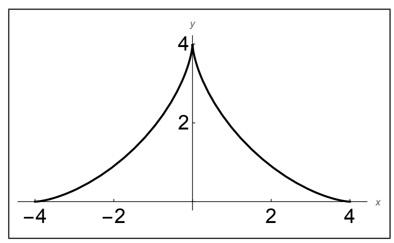
(b)



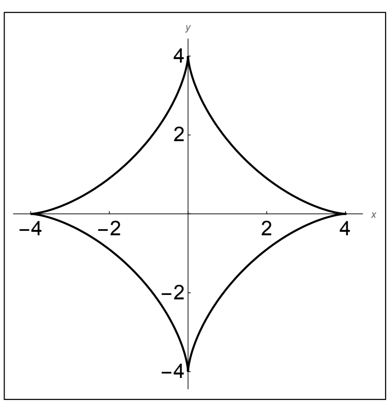
(c)



(d)



(e)



Partial Credit

Please show all of your work for credit in questions 11-13.
If some of the work that you wish to have considered for points is on another page, please indicate where the work is with words and arrows.

11.(13pts) Find the radius of convergence and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{3^n (x+2)^n}{n}.$$

If, in the course of the solution, you test for convergence of a series, please state clearly which test you are using.

Radius Of Convergence: _______ **Interval Of Convergence:**_______

 Initials: ____________

12.(13pts) (a) Find a power series representation (with center 0) for the antiderivative

$$F(x) = \int \frac{1}{1+x^7}\, dx,$$

which satisfies the initial condition: $F(0) = 0$.
Hint: Use your knowledge of a well known series.

(b) Use part (a) to find an expression for the definite integral

$$\int_0^1 \frac{1}{1+x^7}\, dx,$$

as the sum of an infinite series (note: the variable x should not appear in your answer)

(c) Use the alternating series estimation theorem to estimate the value of the above definite integral (in part (b)) so that the error of estimation is less than $\frac{1}{10}$.

13.(13pts) For parts (a) and (b) below, consider the parametric curve defined by

$$x = 1 + \cos(3t) \qquad y = \sin(3t)$$

for $0 \leq t \leq \frac{\pi}{3}$.

(a) Find the arclength of the given curve.
(a formula from the formula sheet should help.)

This Topic(Calculus of Parametric Curves, Section 10.2) in Not on Exam 3 for Spring 2023.
(Section 10.1 is on Ex3 SP 23)

(b) Find the equation of the tangent line to the curve at the point on the curve where $t = \pi/6$.

14.(1pts) You will be awarded this point if you write your section number or class time next to the name of your instructor <u>and</u> you mark your answers on the front page with an X (<u>not</u> an O) . You may also use this page for

ROUGH WORK

12. Initials: ____________

The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta = \ln|\csc\theta - \cot\theta| + C$$